

The climate-extended credit risk model

Josselin Garnier (Ecole Polytechnique)

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Work triggered by discussions with the association Green RWA
(Anne Gruz, Jean-Baptiste Gaudemet)
see [arXiv:2103.03275](https://arxiv.org/abs/2103.03275)

- Objectives:

- Determine the loss distribution of a credit portfolio.
 - The portfolio is made of loans from a large number of borrowers.
 - The loss of the portfolio L is the sum of the random losses of the borrowers.

↔ L is random. We look for the expected and unexpected losses (expectation and quantile).

- Propose a credit risk model which extends the model defined by the Basel Committee to climate (physical and transition) risks.

- Ingredients:

- Credit/climate rating, (IPCC) scenarios.
- Initial loan distribution, reloading of outstanding loans.

Expected loss of the portfolio

- For the q th borrower, the expected loss ($EL^{(q)}$) can be expressed in terms of probability of default ($PD^{(q)}$), loss given default ($LGD^{(q)}$), exposure at default ($EAD^{(q)}$):

$$EL^{(q)} = PD^{(q)} \times LGD^{(q)} \times EAD^{(q)}$$

- The expected loss $L^e = \mathbb{E}[L]$ of the portfolio
 - is the sum of the expected individual losses,
 - can be expressed by grouping the borrowers:

The borrowers belong to different groups $g = 1, \dots, G$, that represent

- geographic regions,
- economic sectors,
- climate risk mitigation and adaptation strategies,
- collateral types.

The borrowers have different ratings $i = 1, \dots, K - 1$ at time 0 (the rating K is default).

Expected loss of the portfolio

Expected loss L^e at the time horizon T :

$$L^e = \sum_{t=1}^T L_t^e,$$

$$L_1^e = \sum_{g=1}^G \sum_{i=1}^{K-1} (M_{g,1})_{iK} \text{LGD}_{g,i,1} \text{EAD}_{g,i,1},$$

$$L_t^e = \sum_{g=1}^G \sum_{i,j=1}^{K-1} (M_{g,1} \cdots M_{g,t-1})_{ij} (M_{g,t})_{jK} \text{LGD}_{g,j,t} \text{EAD}_{g,i,t}, \text{ for } t \geq 2.$$

Here

- $M_{g,t}$: unconditional $K \times K$ migration matrix,
- $\text{EAD}_{g,i,t}$: Exposition At Default, total exposure at default (in case of default at time t) for all borrowers in group g and with initial rating i ,
- $\text{LGD}_{g,j,t}$: Loss Given Default,

depend on group $g \in \{1, \dots, G\}$ and time $t \in \{1, \dots, T\}$.

Expected loss of the portfolio: Migration matrix

initial rating	credit rating at year-end							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	0,9112	0,0800	0,0070	0,0010	0,0005	0,0001	0,0001	0,0001
AA	0,0070	0,9103	0,0747	0,0060	0,0010	0,0007	0,0002	0,0001
A	0,0011	0,0234	0,9154	0,0508	0,0061	0,0026	0,0001	0,0005
BBB	0,0002	0,0030	0,0565	0,8798	0,0475	0,0105	0,0010	0,0015
BB	0,0001	0,0010	0,0055	0,0777	0,8177	0,0795	0,0085	0,0100
B	0,0000	0,0005	0,0025	0,0045	0,0700	0,8350	0,0375	0,0500
CCC	0,0000	0,0001	0,0010	0,0030	0,0259	0,1200	0,6500	0,2000
Default	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	1,0000

1-year migration matrix with $K = 8$.

Each row corresponds to an initial rating.

Each column corresponds to a rating at the end of one year.

As "Default" is absorbing, the last line is of the form $(0, \dots, 0, 1)$.

Unexpected loss of the portfolio: The need for a credit risk model

- The expected loss $L^e = \mathbb{E}[L]$ of the portfolio is the sum of the expected individual losses.
- The unexpected loss is a quantile L^u of the loss of the portfolio:

$$\mathbb{P}(L \leq L^u) = 0.999 \text{ (or 0.99 or 0.9)}$$

The quantile of a sum is not the sum of the quantiles.

↔ A model is needed for the dependence structure.

Asymptotic Single Risk Factor (ASRF) model

The ASRF model

- is a default-mode (Merton-type) model proposed by Vasicek in 1991,
- has played a central role for its regulatory applications in the Basel Capital Accord Framework,
- is based on the following assumptions:
 - 1 a unique systematic risk factor (single-factor model): economic risk
↪ the losses of the borrowers are correlated only through one systematic factor,
 - 2 an infinitely granular portfolio (characterized by a large number of small size loans)
↪ diversification of the idiosyncratic risks, but not of the systematic risk,
 - 3 a dependence structure described by a Gaussian copula
↪ the most important theoretical hypothesis,
- gives closed-form expressions for the expected and unexpected losses.

Asymptotic Single Risk Factor (ASRF) model

- The q th borrower defaults before time t if a latent variable $X_t^{(q)}$ (normalized asset) goes below a threshold value:

$$X_t^{(q)} = a^{(q)} Z_t + \sqrt{1 - (a^{(q)})^2} \varepsilon_t^{(q)}$$

where

- Z_t = systematic (economic) risk factor,
- $\varepsilon_t^{(q)}$ = idiosyncratic factor,
- $a^{(q)} = a_g$ factor loading (Basel: constant; here: depends on group).

Gaussian copula: $(Z_t, \varepsilon_t^{(1)}, \varepsilon_t^{(2)}, \dots)$ are i.i.d. standard Gaussian.

- The threshold values are obtained from the group-dependent unconditional migration matrices

$$z_{g,ij} = \Phi^{-1} \left(\sum_{j'=j}^K (\mathbf{M}_g)_{ij'} \right), \quad \mathbb{P}(X_t^{(q)} \leq z_{g,ij}) = \Phi(z_{g,ij})$$

- The group-dependent conditional migration matrix is

$$\sum_{j'=j}^K (\mathbf{M}_g(Z_t))_{ij'} = \mathbb{P}(X_t^{(q)} \leq z_{g,ij} | Z_t) = \Phi \left(\frac{z_{g,ij} - a_g Z_t}{\sqrt{1 - (a_g)^2}} \right)$$

The Climate-extended model

- is a Multi-Factor Merton-type model,
- is based on the following assumptions:
 - 1 several systematic risk factors (multi-factor model): economic, physical, transition risks,
 - 2 an infinitely granular portfolio (characterized by a large number of small size loans),
 - 3 a dependence structure described by a Gaussian copula,
- gives efficient Monte-Carlo estimations of the expected and unexpected losses.

Basic references:

- Vasicek Model
Vasicek, O., The distribution of loan portfolio value, Risk, Dec. 2002.
- Multi-Factor Merton Model
Pykhtin, M., Multi-factor adjustment, Risk, March 2004.

Additional ingredients (compared to ASRF):

- Idiosyncratic risks, economic risk are stationary.

- Physical and transition risks evolve in time.

↪ *Climate scenarios are needed for the intensities of the systematic risk factors.*

- Physical risk factors can be regional.

- Systematic risk factors can be correlated.

For instance, anti-correlation between economic and transition risks or correlation between regional physical risks.

↪ *Correlation structure between systematic risk factors is needed.*

- Expositions of borrowers to systematic risk factors (micro-correlations) may evolve in time (by mitigation strategies).

↪ *Micro-correlations w.r.t. systematic risk factors are needed for all groups.*

- The historical unconditional migration matrices are used at $t = 0$.

↪ *Same historical migration matrices as for ASRF model are needed.*

Note: The unconditional migration matrices evolve in time due to the non-stationarity of the physical and transition risks.

Climate-Extended Risk Model (CERM) - structure

The q th borrower defaults before time t if a latent variable $X_t^{(q)}$ (normalized asset) goes below a threshold value:

$$X_t^{(q)} = \mathbf{a}_t^{(q)} \cdot \mathbf{Z}_t + \sqrt{1 - \mathbf{a}_t^{(q)} \cdot \mathbf{C} \mathbf{a}_t^{(q)}} \varepsilon_t^{(q)}$$

where

\mathbf{Z}_t = systematic risk factors (with correlation matrix \mathbf{C}),

$\varepsilon_t^{(q)}$ = idiosyncratic factor,

$\mathbf{a}_t^{(q)}$ = factor loadings; they depend on time-dependent macro-correlations and time- and group-dependent micro-correlations.

- macro-correlations: intensities of the systematic risk factors, expressed in same units (impact to GDP growth rate for instance);
 - *constant* for economic risk;
 - given by (IPCC) *carbon emission pathway* for transition risk;
 - given by (IPCC) *GDP growth rate assessment* for physical risk.
- micro-correlations: expositions of borrowers to systematic risk factors;
 - given by *climate ratings*.

Climate-Extended Risk Model (CERM) - results

Conditional loss given the systematic risk factors $\mathbf{Z} = (\mathbf{Z}_1, \dots, \mathbf{Z}_T)$:

$$L(\mathbf{Z}) = \sum_{t=1}^T L_t(\mathbf{Z})$$

$$L_1(\mathbf{Z}) = \sum_{g=1}^G \sum_{i=1}^{K-1} (M_{g,1}(\mathbf{Z}_1))_{iK} \text{LGD}_{g,i,1}(\mathbf{Z}_1) \text{EAD}_{g,i,1}$$

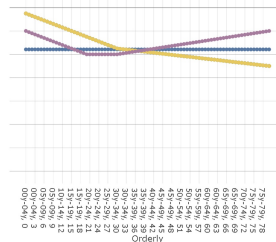
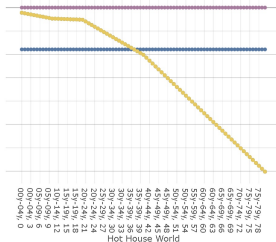
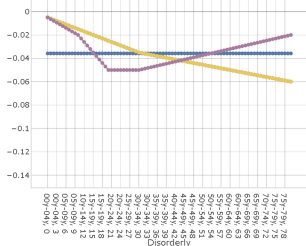
$$L_t(\mathbf{Z}) = \sum_{g=1}^G \sum_{i,j=1}^{K-1} (M_{g,1}(\mathbf{Z}_1) \cdots M_{g,t-1}(\mathbf{Z}_{t-1}))_{ij} (M_{g,t}(\mathbf{Z}_t))_{jK} \text{LGD}_{g,j,t}(\mathbf{Z}_t) \text{EAD}_{g,i,t}$$

for $t \geq 2$. Here

- Explicit formulas are available for all terms.
- $L^e = \mathbb{E}[L(\mathbf{Z})]$.
- L^u such that $\mathbb{P}(L(\mathbf{Z}) \leq L^u) = 99.9\%$ (or 99% or 90%).
- Monte Carlo simulations can be carried out to estimate L^u or the distribution of $L(\mathbf{Z})$.
- Sensitivity indices (w.r.t. groups) can be estimated.

Climate-Extended Risk Model (CERM) - illustrations

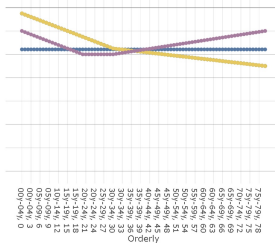
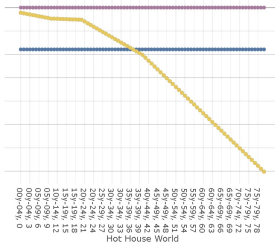
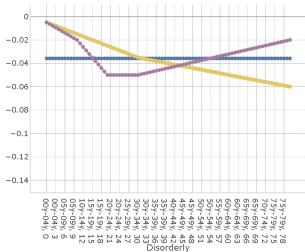
Three climate scenarios (macro-correlations) [IPCC]:



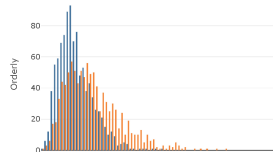
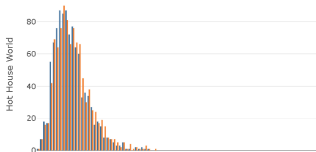
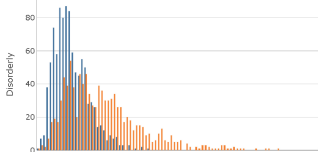
- economic
- physical americas
- physical asia
- physical australia
- physical emea
- physical europe
- transition

[Implementation by Anne Gruz].

Climate-Extended Risk Model (CERM) - illustrations



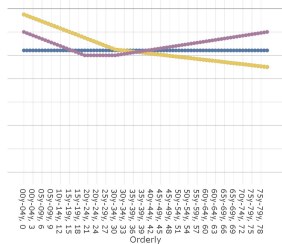
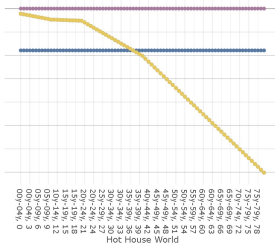
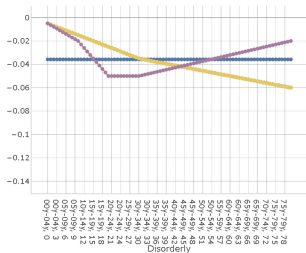
Loss distribution for time horizon $T = 2050$:



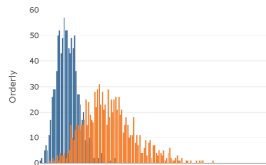
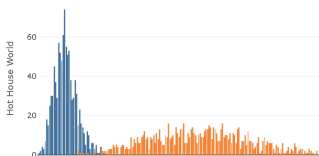
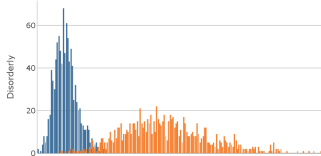
Blue: no physical/transition risk; orange: with physical/transition risks.

[Implementation by Anne Gruz].

Climate-Extended Risk Model (CERM) - illustrations



Loss distribution for time horizon $T = 2100$:



Blue: no physical/transition risk; orange: with physical/transition risks.

[Implementation by Anne Gruz].

Climate-Extended Risk Model (CERM) - summary

- Objectives:
 - Determine the loss distribution of a credit portfolio.
 - Propose a credit risk model which extends the model defined by the Basel Committee to climate (physical and transition) risks.
- Ingredients:
 - Credit/climate rating, (IPCC) scenarios.
 - Initial loan distribution, reloading of outstanding loans.
- Results:
 - Measure the incremental cost of risk and capital to inform credit allocation decisions.
 - Optimize the overall climate strategy, including financing existing clients' adaptation/mitigation plans and shifting assets to green lenders and green collateral.