Bridging socioeconomic pathways of carbon emission and credit risk

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Climate finance, risk and uncertainty modelling

Introduction

- The climate change such as global temperature increase and extreme events related to greenhouse gas emission has become an imminent worldwide challenge.
- IPCC (GIEC in french) Special Reports summarize different potential scenario RCPs (Representative Concentration Pathways) of global warming and related risks.
- The increase of global mean surface temperature by the end of the 2100 is likely to be 0.3^{o}C-1.7^{o}C under RCP2.6; 1.1^{o}C-2.6^{o}C under RCP4.5; 1.4^{o}C-3.1^{o}C under RCP6.0 and 2.6^{o}C-4.8^{o}C under RCP8.5.
- Paris Agreement has set the idealized objective for a global warming around 1.5^{o}C before 2100.
- European Commission planed to cut emissions by 55% by 2030 and become the first climate-neutral continent by 2050.

RCP Projections of greenhouse gas emissions

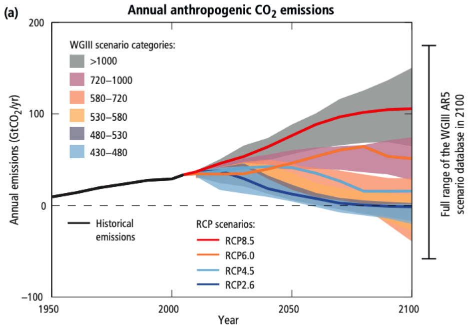

Figure: RCPs describe four different pathways of GHG emissions. Source: Fifth Assessment Report (AR5) of IPCC

- Other possible scenarios, such as Shared Socioeconomic Pathways (SSPs) for CMIP6 project, are developed according to more detailed socio-economic and ecological criteria, for different sectors and countries.

Shared Socioeconomic Pathways (SSPs)

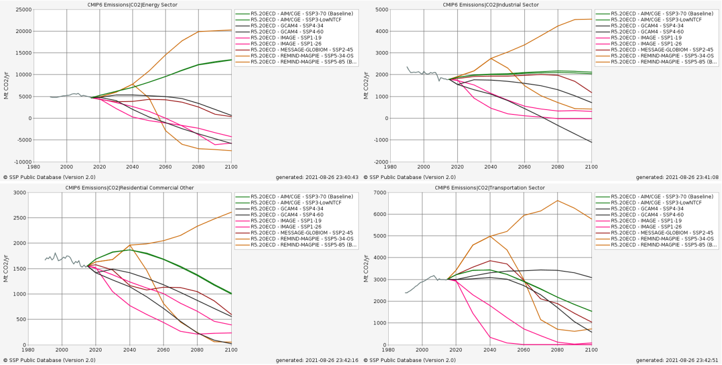

Figure: Historical and scenario-based CO_2 emission, from 1980 to 2100, in Mt/yr in the OECD, according to the activity sectors: Energy (top left), Industry (top right), Residential Commercial (bottom left), Transportation (bottom right).

Outline of our work

- We consider firms who are facing climate transition risks towards a low-carbon production pattern.
- The main objective is to model and quantify how different SSPs projection scenarios of the firm's carbon emission can impact its credit risk.
  - Given an emission scenario, a firm aims to determine its effective emission level under the double criteria of maximizing the production profit and respecting the emission target.
  - The firm's climate-related value process is deduced and the default is modelled by the structural credit model (Merton or Black-Cox): if the value process is not sufficient to cover the debt and liability payment.
  - We compute the default probability related to emission transition and analyse the impact of input SSPs scenarios.

Model Setup

- Let the probability space $(\Omega, \mathcal{A}, \mathbb{P})$ with a filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ represent the market.
- Consider a firm whose production is given by the SDE

$$dP_t = P_t\left(\mu\left(t, P_t, \gamma_t\right)dt + \sigma dW_t\right), \quad P_0 > 0,$$

where $\sigma > 0$ and

  - γ_t is the instantaneous emission rate
  - the function $\mu : (t, x, y) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ satisfies the local Lipschitz condition on x and is of classe C^1 on (x, y)
  - suppose $\partial\mu_x < 0$: overproduction will reduce the production rate and $\partial\mu_y > 0$: empirical studies (e.g. Kalaitzidakis et al. 2018) show that the effect of emissions on production growth is positive.

Emission benchmark

- A series of legislation and policies have been adopted, including European Climate Law and Pact, and the EU Emissions Trading System (ETS).
- Denote by $e_t, t \geq 0$ an emission trajectory for the firm to follow, such as a SSP projection or allocated allowance by EU commission, which will serve as a benchmark of the effective emission γ_t.
- Exceeding the benchmark can induce penalty or losses to the firm such as carbon tax or the cost for purchasing extra allowance through ETS
- Define respectively the cumulative benchmarked and effective emission

$$E_t = \int_0^t e_s ds, \qquad \Gamma_t = \int_0^t \gamma_s ds.$$

  The regulation may apply to the emission trajectory continuously or to the cumulative emission.

Production profit vs emission constraint

- The firm's goal is to maximize its production profit and, at the same time, manage the effective emission by taking into account the advertised constraints.
- The profit function $\pi : \mathbb{R}_+ \to \mathbb{R}$ on the production P_t is increasing and concave, of class C^1, and satisfies the Inada conditions $\lim_{x \to 0+} \pi'(x) = +\infty$ and $\lim_{x \to +\infty} \pi'(x) = 0$
- We consider the regulation constraints by using loss functions related to risk measures by Föllmer and Schied.
- Let $\ell : \mathbb{R} \to \mathbb{R}$ be an increasing and convex loss function with initial value $\ell(0) = 0$ and quadratic growth, i.e., $\ell(x) = \mathcal{O}(|x|^2)$ as $|x| \to +\infty$.

Optimization problem

- Maximize the profit function with emission trajectory constraint

$$J_\infty(\gamma) := \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\pi(P_t) - \mathcal{C}(\gamma_t) - \ell(\gamma_t - e_t)\right) dt\right] \quad (1)$$

  - where $r \geq 0$ is a constant discount rate
  - $\mathcal{C} : \mathbb{R}_+ \to \mathbb{R}_+$ is emission-related cost function which is increasing and convex meaning that higher emissions induce over-usage deterioration.

- Aim to solve

$$\widehat{J} = \sup_{\gamma \in \mathcal{A}} J_\infty(\gamma)$$

where $\mathcal{A}$ is the admissible strategy set such that $\mathbb{E}[(\int_0^\infty \gamma_t dt)^2] < +\infty$, and that for any $x \geq 0$,

$$\int_0^\infty |\mu(t, x, \gamma_t)|^2 \, dt < +\infty, \qquad \text{a.s..}$$

Alternative formulation with finite horizon

- We may consider a final horizon time $T > 0$ such as 2050 or 2100 where an extra cumulative emission penalty is included.
- The objective function becomes

$$J_T(\gamma) := \mathbb{E}\left[\int_0^T e^{-rt}\left(\pi(P_t) - \mathcal{C}(\gamma_t) - \ell_1(\gamma_t - e_t)\right) dt - e^{-rT}\ell_2\left(\Gamma_T - E_T\right)\right]$$

  where ℓ_1 and ℓ_2 are two loss functions
- and we'll solve

$$\widehat{J}_T = \sup_{\gamma \in \mathcal{A}} J_T(\gamma)$$

  where $\mathcal{A}$ is the admissible strategy set such that $\mathbb{E}[\Gamma_T^2] < +\infty$, and that for any $x \geq 0$,

$$\int_0^T |\mu(t, x, \gamma_t)|^2 \, dt < +\infty, \qquad \text{a.s..}$$

Resolution of optimization problems

- These optimisation problems are classical and can be solved by adopting the Pontryagin's maximum principle for the optimal strategy by using the method of Lagrange multipliers applied to a constrained optimization problem.
- Introduce the following change of variables: the log-production $p_t := \log P_t$ which solves

$$dp_t = \overline{\mu}(t, p_t, \gamma_t)dt + \sigma dW_t,$$

with $\overline{\mu}(t, x, y) := \mu(t, e^x, y) - \frac{1}{2}\sigma^2$ and the auxiliary cost function

$$\overline{\pi}(x) := \pi(e^x)$$

Optimal effective emission

- We characterize the solution of the infinite problem $J_\infty(\widehat{\gamma})$.
- Let

$$Y_t = \mathbb{E}\left[\int_t^\infty e^{-ru+\int_t^u \partial_x \overline{\mu}(t,p_s,\gamma_s)ds} \,\overline{\pi}'(p_u)du \middle| \mathcal{F}_t \right]$$

- The optimal effective emission $\widehat{\gamma}$ is then given as the solution of the following equation

$$\mathcal{C}'(\widehat{\gamma}_t) + \ell'(\widehat{\gamma}_t - e_t) = e^{rt}\partial_y \overline{\mu}(t, \widehat{p}_t, \widehat{\gamma}_t)\,\widehat{Y}_t$$

- Note that $\lim_{t\to+\infty} Y_t = 0$.

Optimal emission with finite time horizon

- The finite horizon problem can be solved in a similar way. The difference lies in the extra terminal constraint.
- The solution for $\widehat{J_T} = J_T(\widehat{\gamma})$ is characterized by the following linear BSDE

$$\begin{cases} dY_t^1 & = -\left(e^{-rt}\overline{\pi}'(p_t) + \partial_x \overline{\mu}(t, p_t, \gamma_t)\, Y_t^1\right) dt + dM_t^1, \\ Y_T^1 & = 0 \end{cases}$$

where M^1 is an $\mathbb{F}$-martingale, so that

$$\widehat{Y}_t^1 = \mathbb{E}\left[\int_t^T e^{-ru + \int_t^u \partial_x \overline{\mu}(t, \widehat{p}_s, \widehat{\gamma}_s) ds}\, \overline{\pi}'(\widehat{p}_u) du \middle| \mathcal{F}_t \right]$$

- The optimal emission $\widehat{\gamma}$ satisfies

$$e^{-rt}\left[\mathcal{C}'(\widehat{\gamma}_t) + \ell_1'(\widehat{\gamma}_t - e_t)\right] + \mathbb{E}\left[e^{-rT} \ell_2'\left(\widehat{\Gamma}_T - E_T\right) \middle| \mathcal{F}_t \right] = \partial_y \overline{\mu}(t, \widehat{p}_t, \widehat{\gamma}_t)\, \widehat{Y}_t^1$$

Emission-related credit risk

- Credit risk means the possibility and potential losses due to the incapacity of the firm to reimburse its debt obligations.
- In the structural approach of credit modelling, a firm defaults if its value is not sufficient to repay the debt liability.
- In our setting, we aim to analyse the emission impact on default probability and define the value process of the firm V_t^γ by the so-called "discounted cash flow" approach

$$V_t^\gamma = \mathbb{E}\left[\int_t^\infty e^{-r(u-t)}\left(\pi(P_u) - \mathcal{C}(\gamma_u) - \ell(\gamma_u - e_u)\right) du \middle| \mathcal{F}_t\right].$$

  which is the conditional discounted value of all future cash flows depending on the effective emission γ.
- The firm will produce according to the optimal emission $\widehat{\gamma}$ and the optimal production $\widehat{P}$ from the previous procedure.

Structural default models

- We describe the firm's value at a given date t by the process $V_t^{\widehat{\gamma}}$ which achieves the firm's optimal value as

$$\widehat{V}_t = \operatorname*{ess\,sup}_{\gamma \in \mathcal{A}(t,\nu)} V_t^{\gamma}$$

- The liability value L_t includes the debt payment and will serve as the default barrier .
- Then the default probability in the Merton model is defined as $DP_t = \mathbb{P}(V_t^{\widehat{\gamma}} < L_t)$, closed-form formula can be obtained for certain model specifications. For Black-Cox model, the default probability is path-dependent and given as $DP_t = \mathbb{P}(\exists s \leq t \text{ s.t. } V_s^{\widehat{\gamma}} < L_s)$, the computation is related to the hitting time across curved boundary.

Application with an explicit model

- The earliest firm-specific emission data go back to 2008 with annual frequency. The limited data set motivates to consider a simple linear projection model.
- Consider an explicit log-production model

$$dp_t = \overline{\mu}(t, p_t, \gamma_t)dt + \sigma dW_t,$$

with an affine drift coefficient

$$\overline{\mu}(t, x, y) = a + bx + cy,$$

where

  - $a \geq 0$ corresponds to an average production level
  - $b \leq 0$ is a mean-reverting parameter with the negative sign meaning that over-production may decrease the production ability
  - $c \geq 0$ describes the dependence of the production with respect to emission

Value process with quadratic penalty

- Choose the profit function $\pi(x) = Nx$ where $N > 0$ represents the average price for one unit of production
- The value process $\widehat{V}$ rewrites as

$$\widehat{V}_t = \mathbb{E}\left[\int_t^{\infty} e^{-r(u-t)} \left(N\widehat{P}_u - \mathcal{C}(\widehat{\gamma}_u) - \ell(\widehat{\gamma}_u - e_u) \right) du \middle| \mathcal{F}_t \right]$$

- The cost and penalty functions are given respectively as

$$\mathcal{C}(x) = \frac{x^2}{2} \qquad \text{and} \qquad \ell(x) = \omega \frac{(x_+)^2}{2},$$

  where ω is a positive constant coefficient and the function x_+ denotes $\max(x, 0)$.
- The quadratic penalty means to accentuate higher quantities of over-emission.

Optimal emission

- By results from the infinite horizon optimization and supposing $r - b > 0$, we have

$$\widehat{\gamma}_t = \left(\mathcal{C}'(\cdot) + \ell'(\cdot - e_t)\right)^{-1}\left(c\int_t^{\infty} e^{(b-r)(u-t)}du\right)$$
$$= \min\left\{\frac{c}{r-b}, \frac{1}{1+\omega}\left(\omega e_t + \frac{c}{r-b}\right)\right\}$$

- The critical value

$$\overline{\gamma} := \frac{c}{r-b}$$

  is attained in case without penalty i.e. $\omega = 0$.
- If $e_t \geq \overline{\gamma}$, then the optimal emission is to remain at the constant level $\overline{\gamma}$ (no effort for the company).
- If $e_t < \overline{\gamma}$, meaning that the regulation requires a stricter mitigation plan, then the optimal emission is given as an affine function of the benchmark.

Default probability

- Given the optimal emission $\widehat{\gamma}$, we have the firm's value as

$$V_t^{\widehat{\gamma}} = N \int_t^{\infty} e^{-r(u-t)} \mathbb{E}\big[\widehat{P}_u | \mathcal{F}_t\big] du - \int_t^{\infty} e^{-r(u-t)} (\mathcal{C}(\widehat{\gamma}_u) + \ell(\widehat{\gamma}_u - e_u)) du$$
$$=: h(t, \widehat{p}_t)$$

  where $h(\cdot,\cdot)$ is some deterministic function.
- The default probability rewrites as

$$\mathbb{P}(V_t^{\widehat{\gamma}} \leq L_t) = \mathbb{P}(\widehat{p}_t \leq (h(t,\cdot))^{-1}(L_t))$$
$$= \Phi\left(\frac{(h(t,\cdot))^{-1}(L) - e^{bt} p_0 - m_{t,0}}{\sigma_{t,0}}\right),$$

  where Φ is the c.d.f. of a standard normal random variable and using that $\widehat{p}_t \sim \mathcal{N}(e^{bt} p_0 + m_{t,0}, \sigma^2_{t,0})$.

Numerical illustration

- We illustrate relevant results for the Energy sector.
- The input are SSPs annual historical and future projection of CO2 emissions from 2015 to 2100
- We consider for each sector 5 different emission benchmark scenarios (including 3 baseline scenarios and 2 new pathways) and deduce corresponding default probability.
- The liability boundary L_t is specified as there is no climate impact by

$$\mathbb{P}(\widehat{V}_t^{\mathrm{ref}} \leq L_t) = 1 - e^{-\lambda_{\mathrm{ref}} t},$$

where λ_{ref} is a reference value for default intensity chosen to be 3%, and $\widehat{V}_t^{\mathrm{ref}}$ corresponds to the optimal value without emission constraint, i.e., $\omega = 0$.

Energy sector

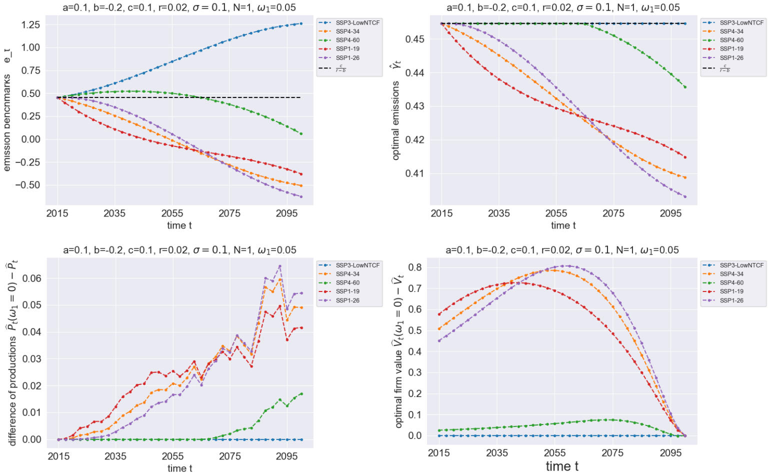

Figure: SSPs emission scenarios e_t up to 2100 (top left), Optimal effective emission $\widehat{\gamma}_t$ (top right), Production difference $\overline{P}_t(\omega = 0) - \widehat{P}_t$ (bottom left), Value process difference $\widehat{V}_t(\omega = 0) - \widehat{V}_t$ (bottom right).

Default probability and intensity for Energy sector

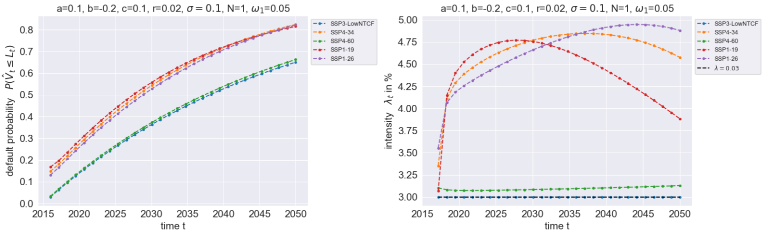

Figure: Default probability up to 2050 (left), Default intensity (right).

- The emission reduction projection has an instantaneous impact on default probability and intensity of the firm:
  - a larger mitigation scenario may imply an increase in the default intensity
  - facing a stricter constraint, the firm chooses to reduce its production and the firm's value decreases accordingly
  - without emission effort, the default intensity remains at the initial level

Default probability under Black-Cox model

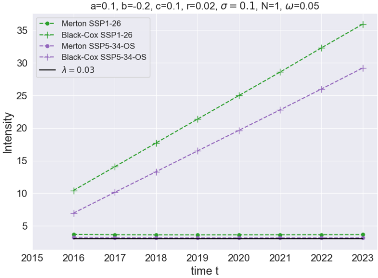

Figure: Default probability in Merton and Black-Cox models respectively for two different SSP scenarios.

- Naturally, the Black-Cox model implies a higher default probability.

Conclusion and Perspective

- We propose a flexible model setup which takes future emission projection pathways as input and compute the the associated default probability as output.
- The model remains quite simple but allows to provide a first answer to analyse quantitatively the impact of climate transition risk on financial credit risk.
- We can extend the default model with more complexe characteristics for example of hybride feature combined with reduced-form credit approach and stochastic intensity parameters.
- We generalize the model to defaultable portfolios with SSPs of different sectors and regions and study the impact on cumulative losses and related risk measures.

Thank you for your attention!