The climate-extended credit risk model

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Work triggered by discussions with the association Green RWA (Anne Gruz, Jean-Baptiste Gaudemet)
see arXiv:2103.03275
Objectives:

- Determine the loss distribution of a credit portfolio.
  - The portfolio is made of loans from a large number of borrowers.
  - The loss of the portfolio $L$ is the sum of the random losses of the borrowers.
  $\rightharpoonup L$ is random. We look for the expected and unexpected losses (expectation and quantile).
- Propose a credit risk model which extends the model defined by the Basel Committee to climate (physical and transition) risks.

Ingredients:

- Credit/climate rating, (IPCC) scenarios.
- Initial loan distribution, reloading of outstanding loans.
Expected loss of the portfolio

- For the $q$th borrower, the expected loss ($\text{EL}(q)$) can be expressed in terms of probability of default ($\text{PD}(q)$), loss given default ($\text{LGD}(q)$), exposure at default ($\text{EAD}(q)$):

$$\text{EL}(q) = \text{PD}(q) \times \text{LGD}(q) \times \text{EAD}(q)$$

- The expected loss $L^e = \mathbb{E}[L]$ of the portfolio
  - is the sum of the expected individual losses,
  - can be expressed by grouping the borrowers:
The borrowers belong to different groups $g = 1, \ldots, G$, that represent
  - geographic regions,
  - economic sectors,
  - climate risk mitigation and adaptation strategies,
  - collateral types.

The borrowers have different ratings $i = 1, \ldots, K - 1$ at time 0 (the rating $K$ is default).
Expected loss of the portfolio

Expected loss $L^e$ at the time horizon $T$:

$$L^e = \sum_{t=1}^{T} L^e_t,$$

$$L^e_1 = \sum_{g=1}^{G} \sum_{i=1}^{K-1} (M_{g,1})_{ik} \text{LGD}_{g,i,1} \text{EAD}_{g,i,1},$$

$$L^e_t = \sum_{g=1}^{G} \sum_{i,j=1}^{K-1} (M_{g,1} \cdots M_{g,t-1})_{ij} (M_{g,t})_{jk} \text{LGD}_{g,j,t} \text{EAD}_{g,i,t}, \text{ for } t \geq 2.$$

Here

- $M_{g,t}$: unconditional $K \times K$ migration matrix,
- $\text{EAD}_{g,i,t}$: Exposition At Default, total exposure at default (in case of default at time $t$) for all borrowers in group $g$ and with initial rating $i$,
- $\text{LGD}_{g,j,t}$: Loss Given Default,

depend on group $g \in \{1, \ldots, G\}$ and time $t \in \{1, \ldots, T\}$. 
### Expected loss of the portfolio: Migration matrix

A 1-year migration matrix with $K = 8$.

Each row corresponds to an initial rating.
Each column corresponds to a rating at the end of one year.
As ”Default” is absorbing, the last line is of the form $(0, \ldots, 0, 1)$.

<table>
<thead>
<tr>
<th>initial rating</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.9112</td>
<td>0.0800</td>
<td>0.0070</td>
<td>0.0010</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>AA</td>
<td>0.0070</td>
<td>0.9103</td>
<td>0.0747</td>
<td>0.0060</td>
<td>0.0010</td>
<td>0.0007</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td>A</td>
<td>0.0011</td>
<td>0.0234</td>
<td>0.9154</td>
<td>0.0508</td>
<td>0.0061</td>
<td>0.0026</td>
<td>0.0001</td>
<td>0.0005</td>
</tr>
<tr>
<td>BBB</td>
<td>0.0002</td>
<td>0.0030</td>
<td>0.0565</td>
<td>0.8798</td>
<td>0.0475</td>
<td>0.0105</td>
<td>0.0010</td>
<td>0.0015</td>
</tr>
<tr>
<td>BB</td>
<td>0.0001</td>
<td>0.0010</td>
<td>0.0055</td>
<td>0.0777</td>
<td>0.8177</td>
<td>0.0795</td>
<td>0.0085</td>
<td>0.0100</td>
</tr>
<tr>
<td>B</td>
<td>0.0000</td>
<td>0.0005</td>
<td>0.0025</td>
<td>0.0045</td>
<td>0.0700</td>
<td>0.8350</td>
<td>0.0375</td>
<td>0.0500</td>
</tr>
<tr>
<td>CCC</td>
<td>0.0000</td>
<td>0.0001</td>
<td>0.0010</td>
<td>0.0030</td>
<td>0.0259</td>
<td>0.1200</td>
<td>0.6500</td>
<td>0.2000</td>
</tr>
<tr>
<td>Default</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
The expected loss $L^e = \mathbb{E}[L]$ of the portfolio is the sum of the expected individual losses.

The unexpected loss is a quantile $L^u$ of the loss of the portfolio:

$$\mathbb{P}(L \leq L^u) = 0.999 \text{ (or 0.99 or 0.9)}$$

The quantile of a sum is not the sum of the quantiles.

$\rightarrow$ A model is needed for the dependence structure.
The ASRF model

- is a default-mode (Merton-type) model proposed by Vasicek in 1991,
- has played a central role for its regulatory applications in the Basel Capital Accord Framework,
- is based on the following assumptions:

1. a unique systematic risk factor (single-factor model): economic risk
   - the losses of the borrowers are correlated only through one systematic factor,

2. an infinitely granular portfolio (characterized by a large number of small size loans)
   - diversification of the idiosyncratic risks, but not of the systematic risk,

3. a dependence structure described by a Gaussian copula
   - the most important theoretical hypothesis,

- gives closed-form expressions for the expected and unexpected losses.
Asymptotic Single Risk Factor (ASRF) model

- The $q$th borrower defaults before time $t$ if a latent variable $X^{(q)}_t$ (normalized asset) goes below a threshold value:

$$X^{(q)}_t = a^{(q)} Z_t + \sqrt{1 - (a^{(q)})^2 \varepsilon^{(q)}_t}$$

where
- $Z_t =$ systematic (economic) risk factor,
- $\varepsilon^{(q)}_t =$ idiosyncratic factor,
- $a^{(q)} = a_g$ factor loading (Basel: constant; here: depends on group).

Gaussian copula: $(Z_t, \varepsilon^{(1)}_t, \varepsilon^{(2)}_t, \ldots)$ are i.i.d. standard Gaussian.

- The threshold values are obtained from the group-dependent unconditional migration matrices

$$z_{g,ij} = \Phi^{-1}\left(\sum_{j' = j}^K (M_g)_{ij'}\right), \quad \mathbb{P}(X^{(q)}_t \leq z_{g,ij}) = \Phi(z_{g,ij})$$

- The group-dependent conditional migration matrix is

$$\sum_{j' = j}^K (M_g(Z_t))_{ij'} = \mathbb{P}(X^{(q)}_t \leq z_{g,ij} | Z_t) = \Phi\left(\frac{z_{g,ij} - a_g Z_t}{\sqrt{1 - (a_g)^2}}\right)$$
The Climate-extended model

- is a Multi-Factor Merton-type model,
- is based on the following assumptions:
  1. several systematic risk factors (multi-factor model): economic, physical, transition risks,
  2. an infinitely granular portfolio (characterized by a large number of small size loans),
  3. a dependence structure described by a Gaussian copula,
- gives efficient Monte-Carlo estimations of the expected and unexpected losses.

Basic references:

- Vasicek Model
- Multi-Factor Merton Model
Additional ingredients (compared to ASRF):

- Idiosyncratic risks, economic risk are stationary.
- Physical and transition risks evolve in time.

→ *Climate scenarios are needed for the intensities of the systematic risk factors.*

- Physical risk factors can be regional.
- Systematic risk factors can be correlated.
  For instance, anti-correlation between economic and transition risks or correlation between regional physical risks.

→ *Correlation structure between systematic risk factors is needed.*

- Expositions of borrowers to systematic risk factors (micro-correlations) may evolve in time (by mitigation strategies).

→ *Micro-correlations w.r.t. systematic risk factors are needed for all groups.*

- The historical unconditional migration matrices are used at $t = 0$.

→ *Same historical migration matrices as for ASRF model are needed.*

Note: The unconditional migration matrices evolve in time due to the non-stationarity of the physical and transition risks.
The $q$th borrower defaults before time $t$ if a latent variable $X_t^{(q)}$ (normalized asset) goes below a threshold value:

\[
X_t^{(q)} = a_t^{(q)} \cdot Z_t + \sqrt{1 - a_t^{(q)} \cdot C a_t^{(q)} \varepsilon_t^{(q)}}
\]

where

- $Z_t =$ systematic risk factors (with correlation matrix $C$),
- $\varepsilon_t^{(q)} =$ idiosyncratic factor,
- $a_t^{(q)} =$ factor loadings; they depend on time-dependent macro-correlations and time- and group-dependent micro-correlations.

- **macro-correlations:** intensities of the systematic risk factors, expressed in same units (impact to GDP growth rate for instance);
  - constant for economic risk;
  - given by (IPCC) carbon emission pathway for transition risk;
  - given by (IPCC) GDP growth rate assessment for physical risk.

- **micro-correlations:** expositions of borrowers to systematic risk factors;
  - given by climate ratings.
Conditional loss given the systematic risk factors $Z = (Z_1, \ldots, Z_T)$:

$$L(Z) = \sum_{t=1}^{T} L_t(Z)$$

$$L_1(Z) = \sum_{g=1}^{G} \sum_{i=1}^{K-1} \left( M_{g,1}(Z_1) \right)_{ik} \cdot \text{LGD}_{g,i,1}(Z_1) \cdot \text{EAD}_{g,i,1}$$

$$L_t(Z) = \sum_{g=1}^{G} \sum_{i,j=1}^{K-1} \left( M_{g,1}(Z_1) \cdot \cdots \cdot M_{g,t-1}(Z_{t-1}) \right)_{ij} \cdot \left( M_{g,t}(Z_t) \right)_{jk} \cdot \text{LGD}_{g,j,t}(Z_t) \cdot \text{EAD}_{g,i,t}$$

for $t \geq 2$. Here

- Explicit formulas are available for all terms.
- $L^e = \mathbb{E}[L(Z)]$.
- $L^u$ such that $\mathbb{P}(L(Z) \leq L^u) = 99.9\%$ (or 99\% or 90\%).
- Monte Carlo simulations can be carried out to estimate $L^u$ or the distribution of $L(Z)$.
- Sensitivity indices (w.r.t. groups) can be estimated.
Three climate scenarios (macro-correlations) [IPCC]:

- Disorderly
- Hot House World
- Orderly

[Implementation by Anne Gruz].
Loss distribution for time horizon $T = 2050$:

Blue: no physical/transition risk; orange: with physical/transition risks.

[Implementation by Anne Gruz].
Loss distribution for time horizon $T = 2100$:

Blue: no physical/transition risk; orange: with physical/transition risks.

[Implementation by Anne Gruz].
Objectives:
- Determine the loss distribution of a credit portfolio.
- Propose a credit risk model which extends the model defined by the Basel Committee to climate (physical and transition) risks.

Ingredients:
- Credit/climate rating, (IPCC) scenarios.
- Initial loan distribution, reloading of outstanding loans.

Results:
- Measure the incremental cost of risk and capital to inform credit allocation decisions.
- Optimize the overall climate strategy, including financing existing clients’ adaptation/mitigation plans and shifting assets to green lenders and green collateral.