Bridging socioeconomic pathways of carbon emission and credit risk

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Climate finance, risk and uncertainty modelling
Introduction

- The climate change such as global temperature increase and extreme events related to greenhouse gas emission has become an imminent worldwide challenge.
- IPCC (GIEC in french) Special Reports summarize different potential scenario RCPs (Representative Concentration Pathways) of global warming and related risks.
- The increase of global mean surface temperature by the end of the 2100 is likely to be $0.3^\circ C - 1.7^\circ C$ under RCP2.6; $1.1^\circ C - 2.6^\circ C$ under RCP4.5; $1.4^\circ C - 3.1^\circ C$ under RCP6.0 and $2.6^\circ C - 4.8^\circ C$ under RCP8.5.
- Paris Agreement has set the idealized objective for a global warming around $1.5^\circ C$ before 2100.
- European Commission planed to cut emissions by 55% by 2030 and become the first climate-neutral continent by 2050.
RCP Projections of greenhouse gas emissions

Figure: RCPs describe four different pathways of GHG emissions. Source: Fifth Assessment Report (AR5) of IPCC

- Other possible scenarios, such as Shared Socioeconomic Pathways (SSPs) for CMIP6 project, are developed according to more detailed socio-economic and ecological criteria, for different sectors and countries.
Shared Socioeconomic Pathways (SSPs)

Figure: Historical and scenario-based CO2 emission, from 1980 to 2100, in Mt/yr in the OECD, according to the activity sectors: Energy (top left), Industry (top right), Residential Commercial (bottom left), Transportation (bottom right).
Outline of our work

- We consider firms who are facing climate transition risks towards a low-carbon production pattern.
- The main objective is to model and quantify how different SSPs projection scenarios of the firm’s carbon emission can impact its credit risk.
  - Given an emission scenario, a firm aims to determine its effective emission level under the double criteria of maximizing the production profit and respecting the emission target.
  - The firm’s climate-related value process is deduced and the default is modelled by the structural credit model (Merton or Black-Cox): if the value process is not sufficient to cover the debt and liability payment.
  - We compute the default probability related to emission transition and analyse the impact of input SSPs scenarios.
Model Setup

- Let the probability space \((\Omega, \mathcal{A}, \mathbb{P})\) with a filtration \(\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}\) represent the market.
- Consider a firm whose production is given by the SDE

\[
dP_t = P_t \left( \mu(t, P_t, \gamma_t) dt + \sigma dW_t \right), \quad P_0 > 0,
\]

where \(\sigma > 0\) and
- \(\gamma_t\) is the instantaneous emission rate
- the function \(\mu : (t, x, y) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}\) satisfies the local Lipschitz condition on \(x\) and is of classe \(C^1\) on \((x, y)\)
- suppose \(\partial \mu_x < 0\): overproduction will reduce the production rate and \(\partial \mu_y > 0\): empirical studies (e.g. Kalaitzidakis et al. 2018) show that the effect of emissions on production growth is positive.
Emission benchmark

- A series of legislation and policies have been adopted, including European Climate Law and Pact, and the EU Emissions Trading System (ETS).
- Denote by $e_t$, $t \geq 0$ an emission trajectory for the firm to follow, such as a SSP projection or allocated allowance by EU commission, which will serve as a benchmark of the effective emission $\gamma_t$.
- Exceeding the benchmark can induce penalty or losses to the firm such as carbon tax or the cost for purchasing extra allowance through ETS.
- Define respectively the cumulative benchmarked and effective emission
  \[ E_t = \int_0^t e_s ds, \quad \Gamma_t = \int_0^t \gamma_s ds. \]

The regulation may apply to the emission trajectory continuously or to the cumulative emission.
Production profit vs emission constraint

- The firm’s goal is to maximize its production profit and, at the same time, manage the effective emission by taking into account the advertised constraints.
- The profit function $\pi : \mathbb{R}_+ \to \mathbb{R}$ on the production $P_t$ is increasing and concave, of class $C^1$, and satisfies the Inada conditions $\lim_{x \to 0^+} \pi'(x) = +\infty$ and $\lim_{x \to +\infty} \pi'(x) = 0$.
- We consider the regulation constraints by using loss functions related to risk measures by Föllmer and Schied.
- Let $\ell : \mathbb{R} \to \mathbb{R}$ be an increasing and convex loss function with initial value $\ell(0) = 0$ and quadratic growth, i.e., $\ell(x) = \mathcal{O}(|x|^2)$ as $|x| \to +\infty$. 
Optimization problem

Maximize the profit function with emission trajectory constraint

\[
J_\infty(\gamma) := \mathbb{E} \left[ \int_0^\infty e^{-rt} (\pi(P_t) - C(\gamma_t) - \ell(\gamma_t - e_t)) \, dt \right] \quad (1)
\]

- where \( r \geq 0 \) is a constant discount rate
- \( C: \mathbb{R}_+ \to \mathbb{R}_+ \) is emission-related cost function which is increasing and convex meaning that higher emissions induce over-usage deterioration.

Aim to solve

\[
\hat{J} = \sup_{\gamma \in \mathcal{A}} J_\infty(\gamma)
\]

where \( \mathcal{A} \) is the admissible strategy set such that

\[
\mathbb{E}[(\int_0^\infty \gamma_t \, dt)^2] < +\infty, \text{ and that for any } x \geq 0,
\]

\[
\int_0^\infty |\mu(t, x, \gamma_t)|^2 \, dt < +\infty, \quad \text{a.s.}
\]
Alternative formulation with finite horizon

- We may consider a final horizon time $T > 0$ such as 2050 or 2100 where an extra cumulative emission penalty is included.
- The objective function becomes

$$J_T(\gamma) := \mathbb{E} \left[ \int_0^T e^{-rt} (\pi(P_t) - C(\gamma_t) - \ell_1(\gamma_t - e_t)) \, dt - e^{-rT} \ell_2 (\Gamma_T - E_T) \right]$$

where $\ell_1$ and $\ell_2$ are two loss functions

- and we’ll solve

$$\hat{J}_T = \sup_{\gamma \in \mathcal{A}} J_T(\gamma)$$

where $\mathcal{A}$ is the admissible strategy set such that $\mathbb{E} \Gamma_T^2 < +\infty$, and that for any $x \geq 0$,

$$\int_0^T |\mu(t, x, \gamma_t)|^2 \, dt < +\infty, \quad \text{a.s.}$$
Resolution of optimization problems

- These optimisation problems are classical and can be solved by adopting the Pontryagin’s maximum principle for the optimal strategy by using the method of Lagrange multipliers applied to a constrained optimization problem.

- Introduce the following change of variables: the log-production $p_t := \log P_t$ which solves

$$dp_t = \bar{\mu}(t, p_t, \gamma_t) dt + \sigma dW_t,$$

with $\bar{\mu}(t, x, y) := \mu(t, e^x, y) - \frac{1}{2}\sigma^2$ and the auxiliary cost function

$$\bar{\pi}(x) := \pi(e^x)$$
Optimal effective emission

- We characterize the solution of the infinite problem $J_\infty(\tilde{\gamma})$.
- Let

$$Y_t = \mathbb{E} \left[ \int_t^\infty e^{-ru+\int_t^u \partial_x \bar{\mu}(t, p_s, \gamma_s) ds} \frac{\pi'(p_u)}{F_u} du \right] \Big| \mathcal{F}_t$$

- The optimal effective emission $\tilde{\gamma}$ is then given as the solution of the following equation

$$C'(\tilde{\gamma}_t) + \ell'(\tilde{\gamma}_t - e_t) = e^{rt} \partial_y \bar{\mu}(t, \tilde{p}_t, \tilde{\gamma}_t) \bar{Y}_t$$

- Note that $\lim_{t \to +\infty} Y_t = 0$. 
Optimal emission with finite time horizon

- The finite horizon problem can be solved in a similar way. The difference lies in the extra terminal constraint.

- The solution for $\hat{J}_T = J_T(\hat{\gamma})$ is characterized by the following linear BSDE

$$\begin{cases}
    dY^1_t = - \left( e^{-rt} \bar{\pi}'(p_t) + \partial_x \bar{\mu}(t, p_t, \gamma_t) Y^1_t \right) dt + dM^1_t, \\
    Y^1_T = 0
\end{cases}$$

where $M^1$ is an $\mathbb{F}$-martingale, so that

$$\hat{Y}^1_t = \mathbb{E} \left[ \int_t^T e^{-ru + \int_t^u \partial_x \bar{\mu}(t, \hat{p}_s, \hat{\gamma}_s) ds} \bar{\pi}'(\hat{p}_u) du \middle| \mathcal{F}_t \right]$$

- The optimal emission $\hat{\gamma}$ satisfies

$$e^{-rt} \left[ C'(\hat{\gamma}_t) + l'_1(\hat{\gamma}_t - e_t) \right] + \mathbb{E} \left[ e^{-rT} l'_2(\bar{T} - E_T) \middle| \mathcal{F}_t \right] = \partial_y \bar{\mu}(t, \hat{p}_t, \hat{\gamma}_t) \hat{Y}^1_t$$
Emission-related credit risk

- Credit risk means the possibility and potential losses due to the incapacity of the firm to reimburse its debt obligations.
- In the structural approach of credit modelling, a firm defaults if its value is not sufficient to repay the debt liability.
- In our setting, we aim to analyse the emission impact on default probability and define the value process of the firm $V_t^\gamma$ by the so-called “discounted cash flow” approach

$$V_t^\gamma = \mathbb{E} \left[ \int_t^\infty e^{-r(u-t)} \left( \pi(P_u) - C(\gamma_u) - \ell(\gamma_u - e_u) \right) du | \mathcal{F}_t \right].$$

which is the conditional discounted value of all future cash flows depending on the effective emission $\gamma$.
- The firm will produce according to the optimal emission $\widehat{\gamma}$ and the optimal production $\widehat{P}$ from the previous procedure.
Structural default models

- We describe the firm’s value at a given date $t$ by the process $\hat{V}_t^\gamma$ which achieves the firm’s optimal value as

$$\hat{V}_t = \operatorname{ess} \sup_{\gamma \in A(t,\nu)} V_t^\gamma$$

- The liability value $L_t$ includes the debt payment and will serve as the default barrier.

- Then the default probability in the Merton model is defined as $DP_t = \mathbb{P}(\hat{V}_t^\gamma < L_t)$, closed-form formula can be obtained for certain model specifications. For Black-Cox model, the default probability is path-dependent and given as $DP_t = \mathbb{P}(\exists s \leq t \text{ s.t. } V_s^\gamma < L_s)$, the computation is related to the hitting time across curved boundary.
Application with an explicit model

- The earliest firm-specific emission data go back to 2008 with annual frequency. The limited data set motivates to consider a simple linear projection model.
- Consider an explicit log-production model

\[ dp_t = \bar{\mu}(t, p_t, \gamma_t) dt + \sigma dW_t, \]

with an affine drift coefficient

\[ \bar{\mu}(t, x, y) = a + bx + cy, \]

where

- \( a \geq 0 \) corresponds to an average production level
- \( b \leq 0 \) is a mean-reverting parameter with the negative sign meaning that over-production may decrease the production ability
- \( c \geq 0 \) describes the dependence of the production with respect to emission
Value process with quadratic penalty

- Choose the profit function $\pi(x) = Nx$ where $N > 0$ represents the average price for one unit of production.
- The value process $\widehat{V}$ rewrites as

$$\widehat{V}_t = \mathbb{E}\left[ \int_t^\infty e^{-r(u-t)} \left( N\widehat{P}_u - C(\widehat{\gamma}_u) - \ell(\widehat{\gamma}_u - e_u) \right) du \middle| \mathcal{F}_t \right]$$

- The cost and penalty functions are given respectively as

$$C(x) = \frac{x^2}{2}$$
and
$$\ell(x) = \omega \frac{(x_+)^2}{2},$$

where $\omega$ is a positive constant coefficient and the function $x_+$ denotes $\max(x, 0)$.
- The quadratic penalty means to accentuate higher quantities of over-emission.
Optimal emission

- By results from the infinite horizon optimization and supposing \( r - b > 0 \), we have

\[
\hat{\gamma}_t = \left( C'(\cdot) + \ell'(\cdot - e_t) \right)^{-1} \left( c \int_t^\infty e^{(b-r)(u-t)} du \right)
= \min \left\{ \frac{c}{r-b}, \frac{1}{1+\omega} \left( \omega e_t + \frac{c}{r-b} \right) \right\}
\]

- The critical value

\[
\overline{\gamma} := \frac{c}{r-b}
\]

is attained in case without penalty i.e. \( \omega = 0 \).

- If \( e_t \geq \overline{\gamma} \), then the optimal emission is to remain at the constant level \( \overline{\gamma} \) (no effort for the company).

- If \( e_t < \overline{\gamma} \), meaning that the regulation requires a stricter mitigation plan, then the optimal emission is given as an affine function of the benchmark.
Default probability

- Given the optimal emission $\hat{\gamma}$, we have the firm’s value as

$$V_t^{\hat{\gamma}} = N \int_t^\infty e^{-r(u-t)} \mathbb{E}[\hat{P}_u | \mathcal{F}_t] du - \int_t^\infty e^{-r(u-t)} (C(\hat{\gamma}_u) + \ell(\hat{\gamma}_u - e_u)) du$$

$$= h(t, \hat{p}_t)$$

where $h(\cdot, \cdot)$ is some deterministic function.

- The default probability rewrites as

$$\mathbb{P}(V_t^{\hat{\gamma}} \leq L_t) = \mathbb{P}(\hat{p}_t \leq (h(t, \cdot))^{-1}(L_t))$$

$$= \Phi \left( \frac{(h(t, \cdot))^{-1}(L) - e^{bt} p_0 - m_{t,0}}{\sigma_{t,0}} \right),$$

where $\Phi$ is the c.d.f. of a standard normal random variable and using that $\hat{p}_t \sim \mathcal{N}(e^{bt} p_0 + m_{t,0}, \sigma_{t,0}^2)$. 


Numerical illustration

- We illustrate relevant results for the Energy sector.
- The input are SSPs annual historical and future projection of CO2 emissions from 2015 to 2100.
- We consider for each sector 5 different emission benchmark scenarios (including 3 baseline scenarios and 2 new pathways) and deduce corresponding default probability.
- The liability boundary $L_t$ is specified as there is no climate impact by
  \[ \mathbb{P}(\hat{V}_t^{\text{ref}} \leq L_t) = 1 - e^{-\lambda_{\text{ref}} t}, \]
  where $\lambda_{\text{ref}}$ is a reference value for default intensity chosen to be 3\%, and $\hat{V}_t^{\text{ref}}$ corresponds to the optimal value without emission constraint, i.e., $\omega = 0$. 
Energy sector

Figure: SSPs emission scenarios $e_t$ up to 2100 (top left), Optimal effective emission $\widehat{\gamma}_t$ (top right), Production difference $\widehat{P}_t(\omega = 0) - \widehat{P}_t$ (bottom left), Value process difference $\widehat{V}_t(\omega = 0) - \widehat{V}_t$ (bottom right).
The emission reduction projection has an instantaneous impact on default probability and intensity of the firm:

- a larger mitigation scenario may imply an increase in the default intensity
- facing a stricter constraint, the firm chooses to reduce its production and the firm’s value decreases accordingly
- without emission effort, the default intensity remains at the initial level
Default probability under Black-Cox model

Figure: Default probability in Merton and Black-Cox models respectively for two different SSP scenarios.

- Naturally, the Black-Cox model implies a higher default probability.
Conclusion and Perspective

- We propose a flexible model setup which takes future emission projection pathways as input and compute the associated default probability as output.

- The model remains quite simple but allows to provide a first answer to analyse quantitatively the impact of climate transition risk on financial credit risk.

- We can extend the default model with more complexe characteristics for example of hybride feature combined with reduced-form credit approach and stochastic intensity parameters.

- We generalize the model to defaultable portfolios with SSPs of different sectors and regions and study the impact on cumulative losses and related risk measures.
Thank you for your attention!